A Novel Use of Kernel Discriminant Analysis as a Higher-Order Side-Channel Distinguisher

Xinping Zhou[1], Carolyn Whitnall[2], Elisabeth Oswald[2], Degang Sun[1] and Zhu Wang[1]

[1]Chinese Academy of Sciences

[2]University of Bristol

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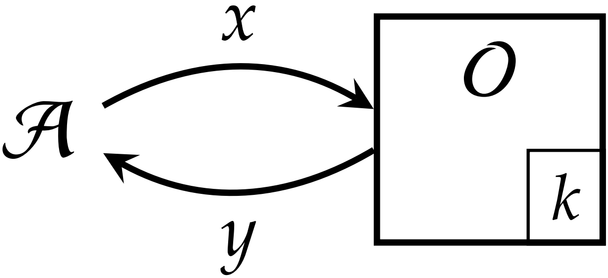
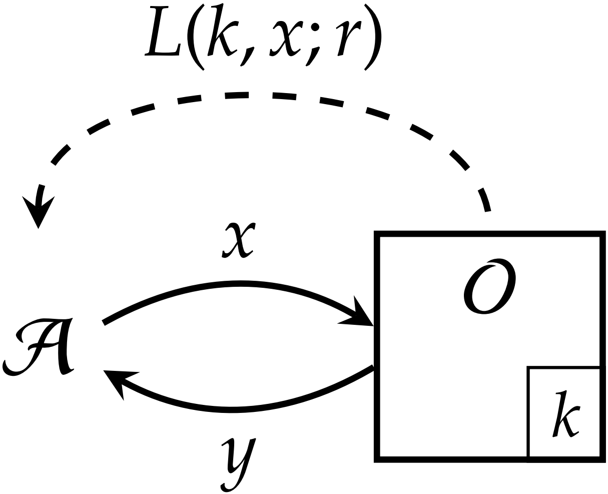
Outline

Introduction

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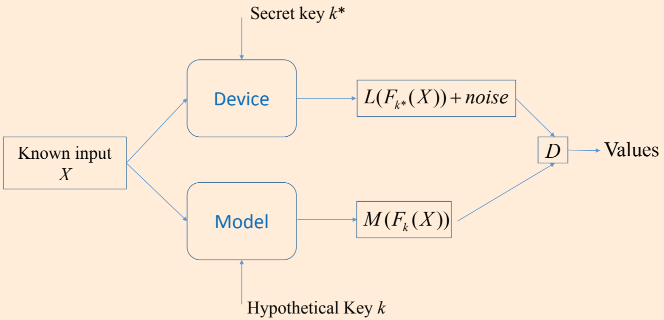
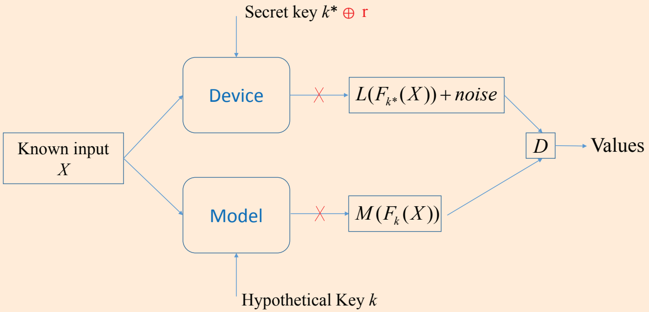
Introduction

Differential side channel



Introduction

Masking countermeasure



Introduction

Logic motivation of this work

- Linear Discriminant Analysis (LDA) was used for fist-order dimensionality reduction. (@ CHES 2008 by Standaert et al.)
- LDA was used as first order distinguisher. (@ RFIDsec 2016 by Mahmudlu et al.)
- Kernel Discriminant Analysis (KDA) was successfully used for dimensionality reduction (or POI selection) in higher-order implementation. (@ CARDIS 2016 by Cagli et al.)
- KDA is proposed as higher-order distinguisher in this work.

- 1. Standaert, F. X., Archambeau, C. (2008). Using subspace-based template attacks to compare and combine power and electromagnetic information leakages. CHES 2008, 411-425.
- 2. Mahmudlu, R., Banciu, V., Batina, L., Buhan, I. (2016). LDA-Based Clustering as a Side-Channel Distinguisher. RFIDsec 2016, 62-75.
- 3. Cagli, Eleonora, C¨¦cile Dumas, and Emmanuel Prouff. Kernel Discriminant Analysis for Information Extraction in the Presence of Masking. CARDIS 2016. 1-22.

Masking countermeasure (boolean masking)

- Sensitive value is to split into several shares

$$s = r_0 \otimes r_1 \otimes ... \otimes r_d$$

- The whole leakages are $\mathbf{l} = (l_0, l_1, ..., l_d)$ with

$$l_0 = L_0 \circ (s \oplus r_1 \oplus \ldots \oplus r_d) + \varepsilon_0$$
$$l_i = L_i \circ (r_i) + \varepsilon_i, \qquad \text{for } 1 \leq i \leq d.$$

Higher-order DPA

$$R^{(d+1)\ell} \xrightarrow{CF} R^{\ell^{d+1}} \xrightarrow{D} k^*$$

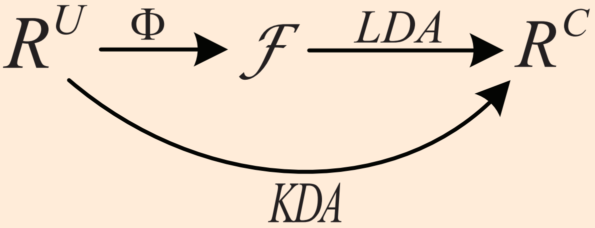
Preliminary

Linear discriminant analysis

- LDA seeks the directions on that the labeled data have max ratio of between-cluster scatter and within-class scatter.
  - LDA is used as reduction tool in profiled-analysis in SCA.
  - Based on the ratio of between-cluster scatter and within-class scatter, it can distinguish the correct key hypothesis and wrong ones.

Preliminary

Linear discriminant analysis with kernels



Kernel discriminant analysis

- KDA seeks the optimal directions in a non-linear space.
  - KDA is used as dimentionality reduction tool in higher-order profiled-analysis in SCA.
  - The eigenvectors with largest eigenvalues are selected in the dimentionality reduction.

Methodology

Natural common ground

$$R^{(d+1)\ell} \xrightarrow{CF} R^{\ell^{d+1}} \xrightarrow{D} k^*$$

$$R^U \xrightarrow{\Phi} \mathcal{F} \xrightarrow{LDA} R^C$$

$$R^U \xrightarrow{KDA} R^C$$

Methodology

Basic idea of KDA distinguisher

- If key hypothesis is correct, the partition of the whole traces based on the intermediate value corresponds with the real partition.
- In this case, it is easy to find the max ratio of between-cluster distance and inner-cluster distance.
- Otherwise, the clusters are difficult to separate.

Methodology

Detailed procedure of KDA distinguisher

- For each key hypothesis $k \in \mathcal{K}$, do the following:
  - Calculate the intermediate value $z_i = F_k(x_i)$ for each plaintext.
  - Map z_i to a power model prediction m_i, given by $M(z_i)$.
  - Compute the between-class scatter matrix $\mathbf{M}$ and the within-class scatter matrix $\mathbf{N}$, and regularize $\mathbf{N}$ by $\mathbf{N} = \mathbf{N} + \mu\mathbf{I}$.
  - Eigen-decompose the matrix $\mathbf{N}^{-1}\mathbf{M}$. Return the largest eigenvalue as the distinguisher score $\mathcal{D}_k$ for k.
- Rank the pairs $(k, \mathcal{D}_k)$ according to $\mathcal{D}_k$.
- Output the key hypothesis k with the largest $\mathcal{D}_k$ as the best guess on the true subkey.

Methodology

Theoretical Rationale

- The effectiveness of the implicit projection.
- The effectiveness of LDA as a distinguisher in the first-order scenario.

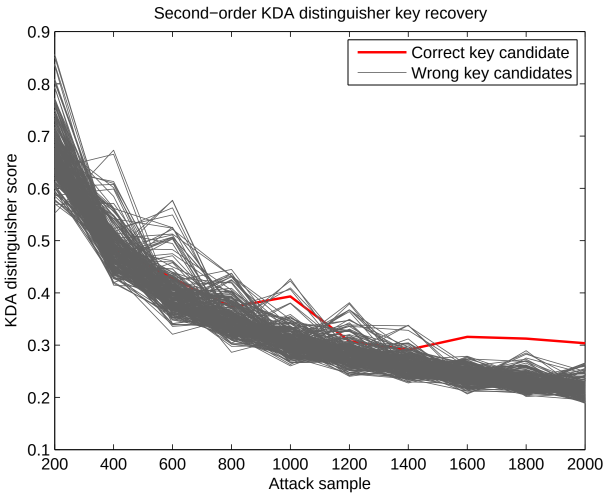
Methodology

Experimental Validation

- Real traces from DPA contest v4 (for second-order analysis).
  - Attack target: XOR result of masked S-box output and masked value of next sub-plaintext in RSM scheme.
- Simulated multivariate leakages (for second-order and third-order analysis).
  - Attack target: XOR result of random shares.
- Kernel function (might not be optimal)
  - The kernel function is $K(\mathbf{x}, \mathbf{y}) = (\mathbf{x} \cdot \mathbf{y})^{d+1}$.
  - Regularization factor $\mu = 100,000$

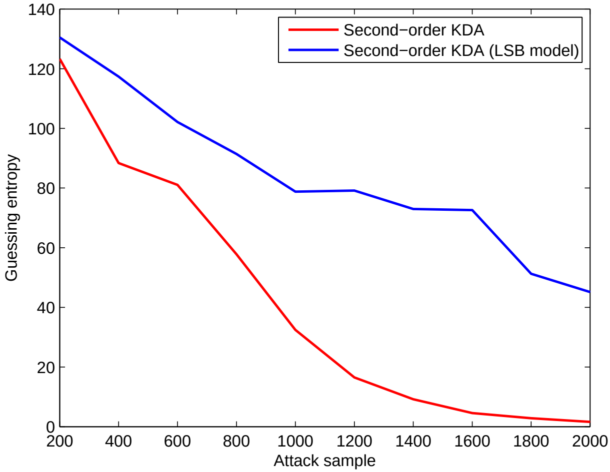
Methodology

KDA on second-order simulated masked implementation with $\sigma = 1$.



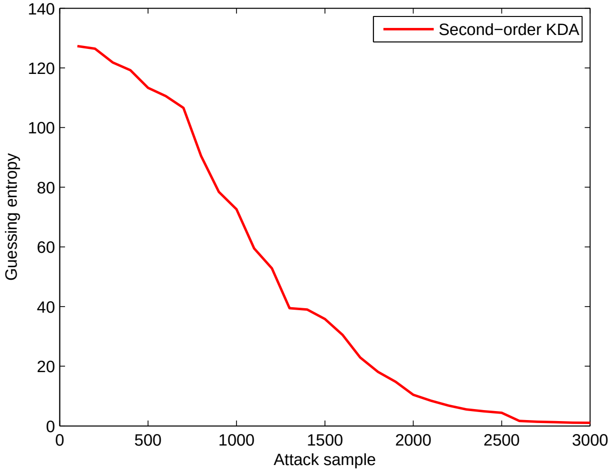
Methodology

KDA on second-order simulated masked implementation with $\sigma = 1$.

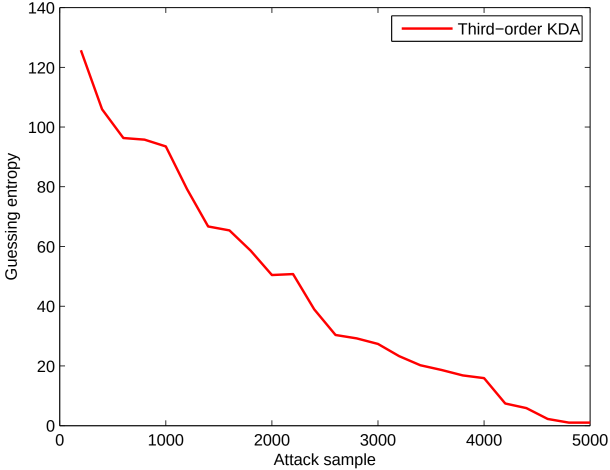


Methodology

Second-order with KDA on DPA v4



Third-order with KDA on simulated masked implementation with $\sigma = 0.01$.



Discussions

Computation Complexity

- Time Complexity:
  - Classical higher-order DPA: $\mathcal{O}(N\ell^{d+1})$.
  - KDA method: $\mathcal{O}(N^2(N + (d + 1)\ell))$.
- Space Complexity:
  - Classical higher-order DPA: $N\ell^{d+1}$.
  - KDA method: $2N^2$.

Discussions

Power Model

- Classical higher-order DPA: Standard proportional power models.
- KDA method: Flexible clustering power models.

Discussions

Limitations and Possibilities

- Classical higher-order DPA using the 'normalised product' combining function with Hamming weight outperforms the KDA .
- It is interesting to deploy the KDA distinguisher in scenarios where higher order correlation DPA is likely to struggle.

Conclusions and Future Perspectives

Conclusions

- Extended KDA for application as distinguisher in masked implementation.
- Showed natural common ground between classical higher-order DPA and KDA.
- Reasoned about the soundness of a KDA-based distinguisher from theoretical perspective and empirically.
- Analyzed the substantial advantages of KDA over higher-order DPA on complexity and power model.

Conclusions and Future Perspectives

Future Perspectives

- Optimizing the parameters such as regularization factor.
- Exploring other kernel functions besides the polynomial function.
- Combining clustering power model in CHES 2015 proposed by Whitnall et al.

- Whitnall, C., Oswald, E.. Robust profiling for DPA-style attacks. CHES 2015. 3-21.

Questions?

Thank you for listening!

Full version available at https://eprint.iacr.org/2017/1051